Angle Modulation

Tomasi (Chapter 6)

Angle Modulation

- **Can modulate carrier's amplitude, frequency, or phase**
- **Last two are modulating carrier's angle**

$$\boxed{\begin{gathered} m(t) = V_c \cos[2\pi f_c t + \theta(t)] \\ \theta(t) = f[v_m(t)] \end{gathered}}$$

Note change in Tomas's notation!
m(t) = v(t)

- **If we modulate $\theta(t)$, it's *phase modulation***
- **If we modulate $\theta'(t)$, , it's *frequency modulation***

Frequency Modulation

- **Example:**
  - **Frequency modulation with sinusoid message**

$$f_{\text{carrier}}(t) = f_c + \Delta f \sin(2\pi f_m t)$$

  - **Curve shows $f_c(t)$**
  - **Note *frequency shift (or deviation)*, Δf**
  - **Rate of frequency change is f_m**
  - **Time domain wave-forms to be done in lab session**

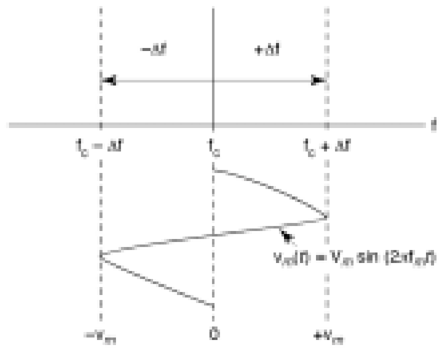

FIGURE 6-1 Angle-modulated wave in the frequency domain. (From Tomasi, 2001, © all rights reserved.)

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Time-Domain Representation

- ***Instantaneous phase* of carrier**

$$IP = 2\pi f_c t + \theta(t)$$

- ***Instantaneous phase deviation: $\theta(t)$***
- ***Instantaneous frequency deviation: $\theta'(t) = d\theta(t)/dt$***
- ***Instantaneous frequency:***

$$2\pi f_i(t) = \frac{d(IP)}{dt} = 2\pi f_c + \theta'(t)$$

$$f_i(t) = f_c + \theta'(t)$$

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Angle Modulation

- **Phase modulation**

$$\theta(t) = Kv_m(t)$$

- ***K*…**
  - **has units of rad/V**
  - **Is called the *deviation sensitivity for phase modulation***

- **Frequency modulation**

$$\theta'(t) = K_1 v_m(t)$$

- ***K*$_1$…**
  - **Has units of (rad/s)/V**
  - **Is called the *deviation sensitivity for frequency modulation***
- **FM is a type of phase modulation if we see that…**

$$\theta(t) = \int \theta'(t)\,dt = K_1 \underbrace{\int v_m(t)\,dt}_{v_{m\ \text{new}}(t)}$$

$$= K_1 v_{m\ \text{new}}(t)$$

Angle Modulation Example Sinusoid Signal

- **Phase modulation**

$$v_m(t) = V_m \cos(2\pi f_m t)$$

$$\theta(t) = Kv_m(t) = KV_m \cos(2\pi f_m t)$$

$$\boxed{m(t) = V_c \cos\left[2\pi f_c t + KV_m \cos(2\pi f_m t)\right]}$$

Note: Tomasi arbitrarily switches to cosines here!

- **Frequency modulation**

$$v_m(t) = V_m \cos(2\pi f_m t)$$

$$\theta(t) = K_1 \int v_m(t)\,dt$$

$$= K_1 V_m \int \cos(2\pi f_m t)\,dt$$

$$= \frac{K_1}{2\pi f_m} V_m \sin(2\pi f_m t)$$

$$\boxed{m(t) = V_c \cos\left[2\pi f_c t + \frac{K_1}{2\pi f_m} V_m \sin(2\pi f_m t)\right]}$$

Table 6.1 on p. 233 summarizes results for both sinusoid message and general message.

Function Plots

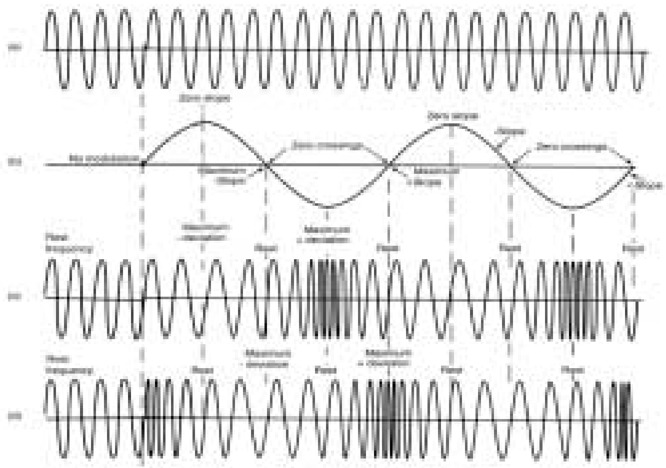

FIGURE 6-3 Phase and frequency modulation of a sine-wave carrier by a sine-wave signal: (a) unmodulated carrier; (b) modulating signal; (c) frequency-modulated wave; (d) phase-modulated wave (From Tomasi, 2001, © all rights reserved.)

- **These message plots will be done in lab using Excel.**

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FM/PM by Sinusoid Message

- **With sinusoid message, both FM and PM "can be written as"…**

"modulation index"

$$m(t) = V_c \cos\left[2\pi f_c t + \begin{array}{c} m\cos(2\pi f_m t) \\ \text{or} \\ \underbrace{m\sin(2\pi f_m t)}_{\theta(t)} \end{array}\right]$$

- ***m* is the *modulation index* or the *peak deviation* [radians]**
  - **Phase Modulation**

    $$\boxed{m = KV_m}$$

  - **Frequency Modulation**

    $$\boxed{m = \frac{K_1 V_m}{2\pi f_m}}$$

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Frequency or Phase Deviation

- **FM**
  - **Δf is the (peak) *frequency deviation* of the FM wave**
  - **$2\Delta f$ is peak-to-peak frequency deviation**
  - **m is ratio of Δf to f_m**
- **with sinusoid message**

$$\Delta f = K_1 V_m$$

$$m = \frac{\Delta f}{f_m}$$

$$m(t) = V_c \cos\left[2\pi f_c t + \underbrace{\frac{\Delta f}{f_m}}_{m} \sin(2\pi f_m t)\right]$$

Typo on earlier notes. Was "cos"

- **PM**
  - **$\Delta\theta$ is the (peak) *phase deviation* of the PM wave**
  - **m is $\Delta\theta$**
- **with sinusoid message**

$$\Delta\theta = K V_m$$

$$m \equiv \Delta\theta$$

$$m(t) = V_c \cos\left[2\pi f_c t + \underbrace{\Delta\theta}_{m} \cos(2\pi f_m t)\right]$$

See Table 6.2 for complete summary and contrast of equations.

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Percent Modulation for FM/PM

- **Compares frequency deviation of actual transmitted wave with the maximum allowed (by law)**

$$\boxed{\%\ \text{modulation} = \frac{\Delta f_{\text{actual}}}{\Delta f_{\text{max allowed}}}}$$

- **E.g., in US $\Delta f_{\text{max allowed}}$ is ±75 kHz around carrier frequency for commercial FM broadcast**
- **If FM signal has frequency deviation of ±37.5 kHz , percent modulation is 50%.**

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PM and FM Modulators

- PM Modulator
  - Directly change phase
  - Unmodulated carrier at "rest frequency"
  - Can be differentiator followed by FM modulator (indirect PM)
- FM modulator
  - Directly changes frequency of carrier (e.g., "voltage controlled oscillator")
  - Can be integrator followed by PM (indirect FM)

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Frequency Domain Representation of FM/PM

- Hard to make statements about general message
- Begin with sinusoid message
  - Will see that spectrum is infinite set of harmonics of $2\pi(f_c \pm nf_m)$
  - Higher harmonics ***(i,.e., large n)*** are negligible

$$m(t) = V_c \cos\left[2\pi f_c t + \underbrace{m\cos(2\pi f_m t)}_{\theta(t)}\right]$$

- Find Fourier series expansion of ***m(t)…***

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Frequency Domain Representation of FM/PM

- Bessel function identity…

$$\cos(\alpha + m\cos\beta) = \sum_{n=-\infty}^{\infty} J_n(m)\cos\left(\alpha + n\beta + \frac{n\pi}{2}\right)$$

- ***$J_n(m)$*** is Bessel function of the first kind ("J") of the order, ***n***, evaluated at ***m***
  - Evaluate using Excel (for example)
- For our function…

$$\alpha = 2\pi f_c t \quad \text{and} \quad \beta = 2\pi f_m t$$

- So…

Frequency Domain Representation of FM/PM

- Frequency components at: ***$0, f_c \pm f_m, f_c \pm 2f_m, \ldots, f_c \pm nf_m, \ldots$***
  - Infinite set of side frequencies centered around carrier
- Amplitude: ***$V_c J_n(m)$***
  - Eventually ***$J_n(m)$*** is negligibly small
  - Note that

    $J_{-n}(m) = (-1)^n J_n(m)$
- Phase: shift of $\pm n\pi/2$ added to each phase

$$m(t) = V_c \sum_{n=-\infty}^{\infty} J_n(m)\cos\left(2\pi f_c t + n2\pi f_m t + \frac{n\pi}{2}\right)$$

$$\boxed{= V_c \sum_{n=-\infty}^{\infty} J_n(m)\cos\left[2\pi(f_c + nf_m)t + \frac{n\pi}{2}\right]}$$

$$= V_c J_0(m)\cos[2\pi f_c t]$$

$$+V_c J_1(m)\cos\left[2\pi(f_c + f_m)t + \frac{\pi}{2}\right]$$

$$+V_c \underbrace{J_{-1}(m)}_{J_{-n}=(-1)^n J_n} \underbrace{\cos\left[2\pi(f_c - f_m)t - \frac{\pi}{2}\right]}_{\cos[x-(\pi/2)]=\sin(x)}$$

$$+V_c J_2(m)\underbrace{\cos\left[2\pi(f_c + 2f_m)t + \underbrace{\frac{2\pi}{2}}_{\pi}\right]}_{\cos(x+\pi)=\cos(x)}$$

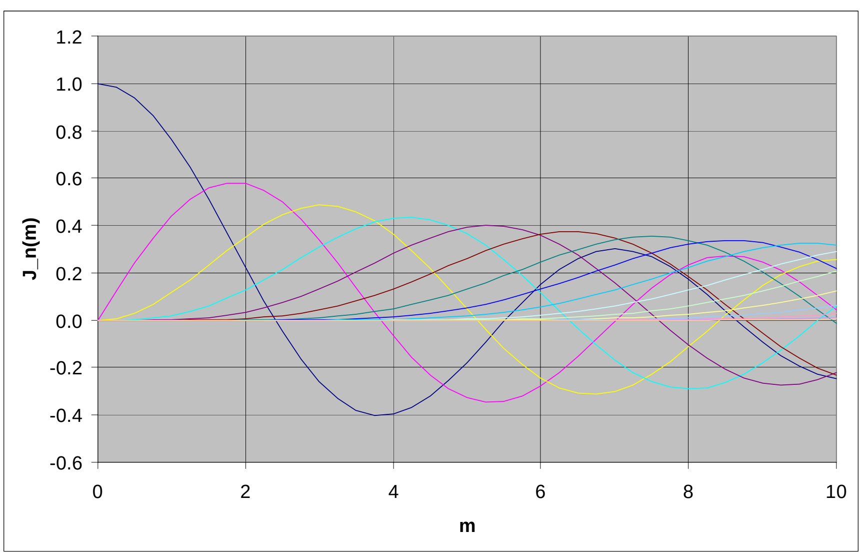
…

$$\boxed{\begin{gathered} +V_c \underbrace{J_{-n}(m)}_{J_{-n}=(-1)^n J_n} \cos\left[2\pi(f_c - nf_m)t - \frac{n\pi}{2}\right] \\ +V_c J_n(m)\cos\left[2\pi(f_c + nf_m)t + \frac{n\pi}{2}\right] \end{gathered}}$$

Bessel Function Table

	J_n															
m	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
0	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.25	0.9844	0.1240	0.0078	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.5	0.9385	0.2423	0.0306	0.0026	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.7	0.8812	0.3290	0.0588	0.0069	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7652	0.4401	0.1149	0.0196	0.0025	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.5118	0.5579	0.2321	0.0610	0.0118	0.0018	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.2239	0.5767	0.3528	0.1289	0.0340	0.0070	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.4	0.0025	0.5202	0.4310	0.1981	0.0643	0.0162	0.0034	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.5	-0.0484	0.4971	0.4461	0.2166	0.0738	0.0195	0.0042	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	-0.2601	0.3391	0.4861	0.3091	0.1320	0.0430	0.0114	0.0025	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	-0.3971	-0.0660	0.3641	0.4302	0.2811	0.1321	0.0491	0.0152	0.0040	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
5	-0.1776	-0.3276	0.0466	0.3648	0.3912	0.2611	0.1310	0.0534	0.0184	0.0055	0.0015	0.0004	0.0001	0.0000	0.0000	0.0000
5.45	-0.0240	-0.3438	-0.1022	0.2688	0.3981	0.3156	0.1809	0.0828	0.0318	0.0106	0.0031	0.0008	0.0002	0.0000	0.0000	0.0000
6	0.1506	-0.2767	-0.2429	0.1148	0.3576	0.3621	0.2458	0.1296	0.0565	0.0212	0.0070	0.0020	0.0005	0.0001	0.0000	0.0000
7	0.3001	-0.0047	-0.3014	-0.1676	0.1578	0.3479	0.3392	0.2336	0.1280	0.0589	0.0235	0.0083	0.0027	0.0008	0.0002	0.0001
8	0.1717	0.2346	-0.1130	-0.2911	-0.1054	0.1858	0.3376	0.3206	0.2235	0.1263	0.0608	0.0256	0.0096	0.0033	0.0010	0.0003
8.65	0.0010	0.2716	0.0618	-0.2430	-0.2303	0.0300	0.2650	0.3376	0.2815	0.1830	0.0994	0.0467	0.0195	0.0073	0.0025	0.0008
9	-0.0903	0.2453	0.1448	-0.1809	-0.2655	-0.0550	0.2043	0.3275	0.3051	0.2149	0.1247	0.0622	0.0274	0.0108	0.0039	0.0013
10	-0.2459	0.0435	0.2546	0.0584	-0.2196	-0.2341	-0.0145	0.2167	0.3179	0.2919	0.2075	0.1231	0.0634	0.0290	0.0120	0.0045

Bessel Function Plots

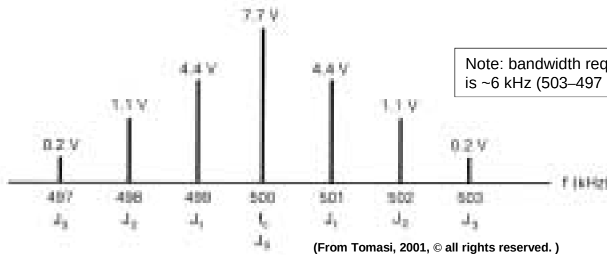


Example 6.2 p. 239

- FM, *m* = 1, message and carrier

$$v_m(t) = V_m \sin[2\pi(1{,}000)t]$$
$$v_c(t) = 10\sin[2\pi(500{,}000)t]$$

- *m*=1, read *m*=1 row of Table 6.3 -> dc + ~3 harmonics
- Values: dc = 0.77V_c=7.7, 0.44V_c, 0.11V_c, 0.02V_c, others are negligible
- Spectrum



Note: bandwidth required is ~6 kHz (503–497 kHz).

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Bandwidth of FM/PM Waveforms

- Channel bandwidth required to send FM/PM waves
- Theoretically infinite; but higher components have negligible amplitudes
- $B_{FM} > B_{AM}$
- $B = f(m, f_m)$ but no simple expression exists
- Exact method:
  - Given *m*, calculate $J_n(m)$ until values are negligible (note *n* of last non-negligible $J_n(m)$)

$$\boxed{B \approx 2n_{\text{non-negligible}} f_m}$$

- Carson's "Rule" (estimate): $\boxed{B_{\text{FM}} \approx 2(\Delta f + f_m)}$
  - Too small; useful estimate of lower bound

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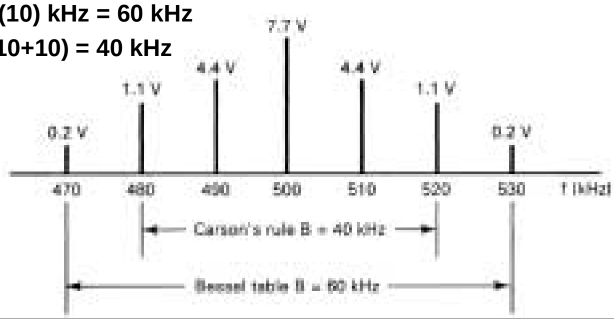
Example 6.3 p. 243

- FM, Δf = 10 kHz, message f_m = 10 kHz with 10 V amplitude; carrier f_c = 500 kHz

$$m = \frac{10 \text{ kHz}}{10 \text{ kHz}} = 1.0$$

- m=1, read m=1 row of Table 6.3 -> dc + ~3 harmonics
- Values: dc = $0.77V_c$=7.7, $0.44V_c$, $0.11V_c$, $0.02V_c$
- Spectrum:
  - Exact method B = 2(3)(10) kHz = 60 kHz
  - Carson's rule: B = 2(10+10) = 40 kHz



Narrow-Band FM

- *Low-index FM*: Small *m* *(m<1)*
  - 1^{st} set of sidebands are only ones non-negligible,
  - $B_{low\text{-}index} = 2f_m$
  - Sometimes called *narrow-band FM* (has minimum bandwidth)
- *High-index FM*: large *m* ($m \geq 10$)
  - B ≈ 2 Δf
- *Medium-index FM*: 1 < m < 10
  - Use exact method (or Carson's rule)

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Deviation Ratio

- **Maximum bandwidth (worst-case) is required when you combine the maximum frequency deviation with the maximum modulation frequency**
- **E.g., by Carson's rule**

$$B_{max} \approx 2\left(\Delta f_{max} + (f_m)_{max}\right)$$

- **Deviation ratio (figure of merit for bandwidth of FM)**

$$\boxed{\mathrm{DR}\,(= m) \equiv \frac{\Delta f_{\max}}{(f_m)_{\max}}}$$

Note: we will also use "DR" for dynamic range!

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Commercial Broadcast AM

- **88-108 MHz band (2 MHz width)**
- **200 kHz per station (100 stations in band)**
- **Maximum frequency deviation: 75 kHz**
- **Maximum modulating frequency: 15 kHz**

- **Worst case deviation ratio would combine maximum frequency deviation with maximum frequency**

$$\mathrm{DR} = m = \frac{\Delta f_{max}}{(f_m)_{max}} = \frac{75}{15} = 5$$

- ***m* = 5 has 8 significant sidebands, so bandwidth is**

$$B_{\max} \approx 2(8)(15) = 240\ \mathrm{kHz}$$

- **B_{max} > 200 kHz allowed; some spillover of 7^{th} and 8^{th} sidebands into adjacent slot**
  - **"Interchannel interference"**
  - **FCC assigns only every other slot**

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Power in FM/PM wave

- *All* power is in constant-amplitude modulated carrier
- ***Average power in unmodulated carrier***: $P_{c\ \text{average}} = V_c^2 / 2R$
- ***Average power in modulated carrier*** is proven (in text) to be the same: $\boxed{P_{t\ \text{average}} = V_c^2 / 2R}$
- ***Total power in modulated carrier*** is sum of power in carrier and significant sidebands…

$$\boxed{\begin{aligned} P_t &= P_0 + P_1 + P_2 + P_3 + \ldots + P_n \\ &= \frac{V_c^2}{2R} + \frac{2V_1^2}{2R} + \frac{2V_2^2}{2R} + \frac{2V_3^2}{2R} + \ldots + \frac{2V_n^2}{2R} \end{aligned}}$$

Noise and Angle Modulation

- **Thermal noise** (a "white noise") adds to FM signal as an interference voltage, $V_n(t)$

Be careful of notation change here! "n" stands for "noise"; it is *not* an integer.

- **Also can be decomposed into sinusoids, f_n**
- **Effect depends on relative size of V_n to V_c**
- **Spectrum of detected noise…**
  - PM demod: constant
  - FM demod: linearly increasing (slope is function of *m*)

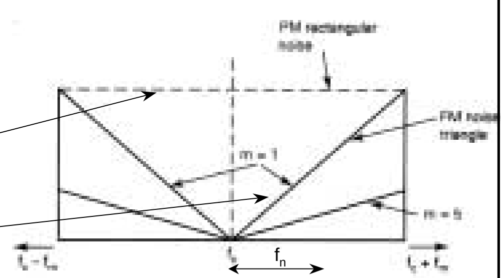


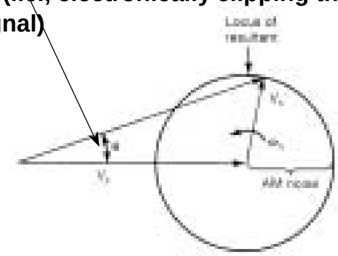
FIGURE 6-10 FM noise triangle (From Tomasi, 2001, © all rights reserved.)

Noise and Angle Modulation

- Consider single-frequency noise voltage
- Amplitude noise becomes phase noise…
- Peak phase deviation due to noise :

$$\theta_{\text{peak}} = \tan^{-1}\left(\frac{V_n}{V_c}\right) \approx \frac{V_n}{V_c} \qquad \text{for } V_n/V_c \ll 1.$$

- Noise effects are lessened by limiting signal voltage value (i.e., electronically clipping the top and bottoms of the received signal)



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Noise and Angle Modulation

- Noise effects adds small FM to signal…

$$\theta(t) = \frac{V_n}{V_c}\sin(2\pi f_n t + \phi_n) \qquad \theta'(t) = 2\pi\Delta f(t) = \left(2\pi f_n \frac{V_n}{V_c}\right)\cos(2\pi f_n t + \phi_n)$$

for $V_n/V_c << 1$

- Peak frequency deviation due to the noise is…

$$2\pi\Delta f_n = \left(\frac{V_n}{V_c}\right)2\pi f_n \;\Rightarrow\; \Delta f_n = \underbrace{\left(\frac{V_n}{V_c}\right)}_{m} f_n = m f_n \qquad \text{for } V_n/V_c \ll 1.$$

- SNR at demodulator output ($V_{detected} \sim \Delta f$)…

$$\text{SNR} = 20\log\left(\frac{V_{s\ \text{detected}}}{V_{N\ \text{detected}}}\right) = 20\log\left(\frac{\Delta f_{\text{signal}}}{\Delta f_{\text{noise}}}\right)$$

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Example 6-6, p. 250

- V_c = 6 cos[2π(110 MHz)t]; Δf_c = 75 kHz;
- V_n = 0.3 cos[2π(109.985 MHz)t]

(a) Frequency of demodulated interference signal:

$$f_{\text{interference}} = f_c - f_n = 110 - 109.985 = 15 \text{ kHz}$$

(b) Peak phase and freq deviations due to interference

$$\Delta\theta_{\text{peak}} = \frac{V_n}{V_c} = \frac{0.3}{6} = 0.05 \text{ radians } = 50 \text{ mr.}$$

$$\Delta f_{\text{peak}} = \frac{V_n}{V_c} f_m = \left(\frac{0.3}{6}\right)(15) = 0.75 \text{ kHz} = 750 \text{ Hz.}$$

(c) SNR at demod output and dB SNR improvement...

$$\text{SNR}_{in} = \frac{V_c}{V_n} = \frac{6}{0.3} = 20 \Rightarrow 26 \text{ dB}$$

$$\text{SNR}_{out} = \frac{\Delta f_{\text{signal}}}{\Delta f_{\text{noise}}} = \frac{75 \text{ kHz}}{0.75 \text{ kHz}} = 100 \Rightarrow 40 \text{ dB}$$

$$\text{dB-improvement} = 40 - 26 = 14 \text{ dB.}$$

FM Preemphasis and Deemphasis

- FM only
- Noise spectrum has triangle shape
- SNR varies with frequency offset

- Emphasize high frequencies over low at XMTR
- SNR is constant
- Reverse (undo) emphasis in RCVR

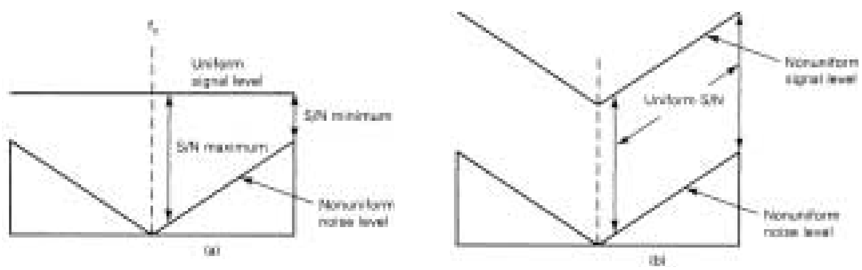
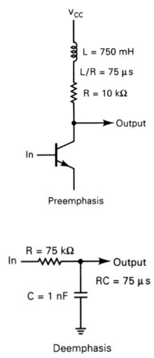
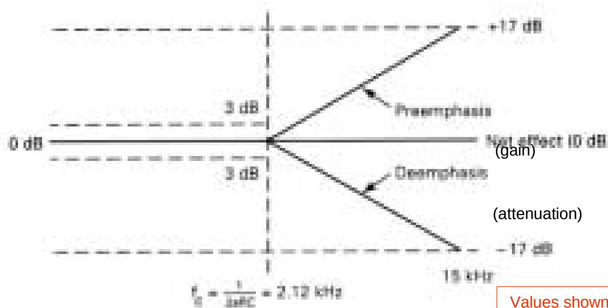


FIGURE 6-12 FM signal-to-noise: (a) without preemphasis; (b) with preemphasis (From Tomasi, 2001, © all rights reserved.)

FM Preemphasis and Deemphasis

- **Circuits and their frequency response…**





Values shown for US FM broadcast.

$$f_{b\ \text{emphasis}} = R/2\pi L \qquad f_{b\ \text{deemphasis}} = 1/2\pi RC$$

$$\text{Set } f_{b\ \text{emphasis}} = f_{b\ \text{deemphasis}}$$

FIGURE 6-13 Preemphasis and deemphasis: (a) schematic diagrams; (b) attenuation curves (From Tomasi, 2001, © all rights reserved.)

- **Peculiar consequence: FM below f_b and PM above f_b**

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Direct and Indirect Angle Modulation

- **FM**
  - **Direct: message modulates frequency directly**
    - **Pro: easy to implement, large $\Delta\phi$, large *m***
    - **Con: hard to maintain carrier freq stability w/o using extra frequency control**
  - **Indirect: PM modulator preceded by integrator**
    - **Pro: can use crystal stabilized carrier**
- **PM**
  - **Direct: message modulates phase directly**
    - **Pro: can use crystal stabilized carrier**
    - **Con: cannot achieve high *m*, large $\Delta\theta_{peak}$**
  - **Indirect: FM modulator preceded by a differentiator**
    - **Pro: can achieve high *m*, large $\Delta\theta_{peak}$**

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Example of Direct FM

- **Linear IC: voltage controlled oscillator (VCO)**

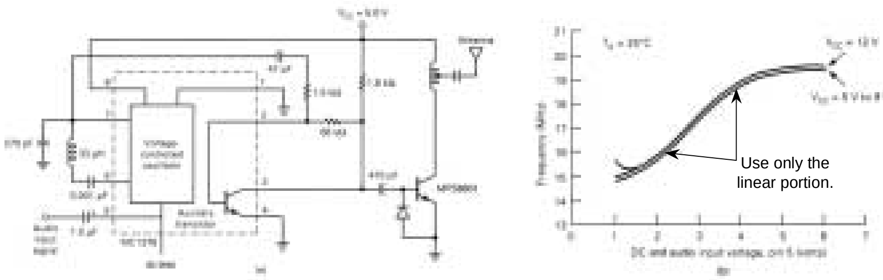

FIGURE 6-19 MC1376 FM transmitter LIC: (a) schematic diagram; (b) VCO output-versus-input frequency-response curve (From Tomasi, 2001, © all rights reserved.)

Example of Direct FM

- **Functional block diagram**

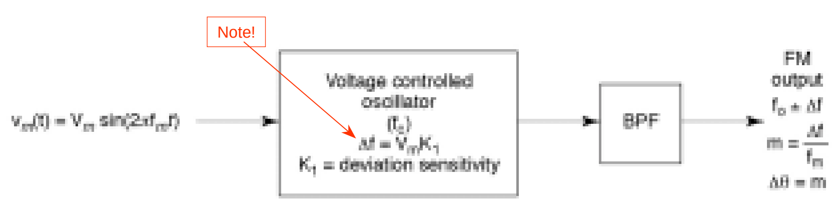

FIGURE 6-18 LIC Direct FM modulator-simplified block diagram (From Tomasi, 2001, © all rights reserved.)

- **Problems: f_c drifts with time and temperature**
  - **FCC requires no drift bigger than ±2 kHz for commercial FM**
  - **Solved with *automatic frequency control* (AFC)**

Frequency Shifts via Upconversion and Multiplication

- **Frequent need to shift frequency from f_c to another frequency** (e.g., another "slot" in the spectrum)
- **Done by…**
  - Heterodyning
  - Frequency multiplication circuits
  - Both

Frequency Shifts via Upconversion and Multiplication

- **Conventional heterodyning**
  - **Changes *fc***
  - **No change to Δf, m, f_m (sideband spacing), B**

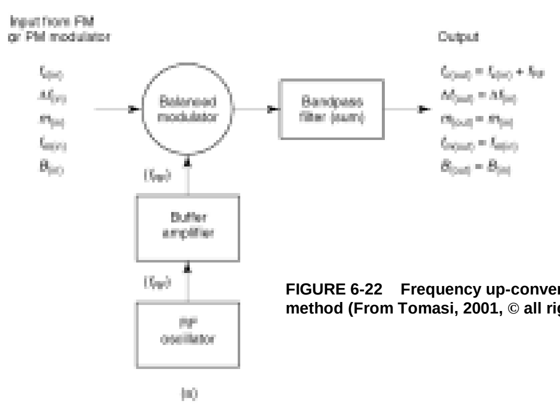

FIGURE 6-22 Frequency up-conversion: (a) heterodyne method (From Tomasi, 2001, © all rights reserved.)

Frequency Shifts via Upconversion and Multiplication

- **Frequency multiplication circuit**
  - **Changes f_c, Δf, m** (increase all by *N*), ***B***
  - **No change to f_m** (sideband spacing)
  - **Since *m* changes, number of sidebands changes** (spacing stays the same), **as does *B***

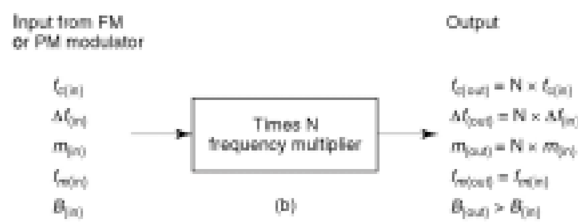

FIGURE 6-22 Frequency up-conversion: (b) multiplication method (From Tomasi, 2001, © all rights reserved.)

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Crosby Direct FM Transmitter

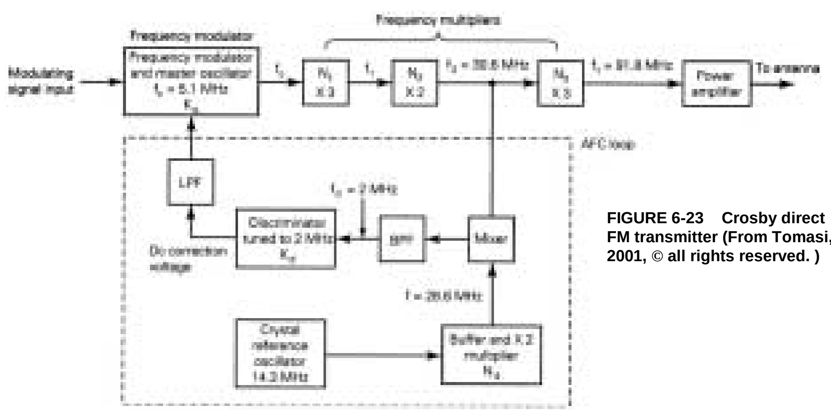

FIGURE 6-23 Crosby direct FM transmitter (From Tomasi, 2001, © all rights reserved.)

- **VCO, frequency multiplication** (18x), **amplification**
- **f_t = (18)(5.1) = 91.8 MHz**
- **$\Delta f_{VCO} = \Delta f_{max}/18$ = 75/18 = 4.167 kHz; $m_{VCO} = 4.167/f_m$ = 4.167/15 = 0.278; $m_{out} = 18\, m_{VCO}$ = (18)(0.278) = 5**

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Direct FM Transmitter

- **Problem: VCO frequency drift exceeds FCC spec**
- **AFC loop: compares f_2 (30.6 MHz) with doubled reference (28.6 MHz), mixer generates sum & difference, BPF difference (2 MHz); apply to discriminator (f-to-v converter; f_d of 2 MHz is 0 V); LPF and pass to VCO as a dc voltage (message is ac only)**

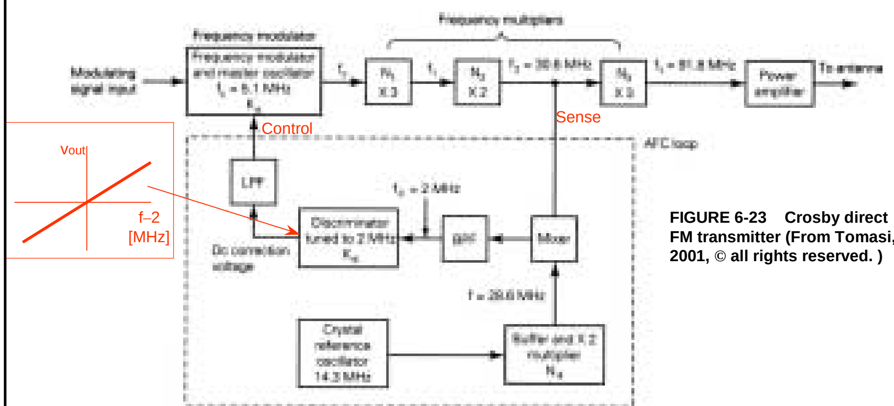

FIGURE 6-23 Crosby direct FM transmitter (From Tomasi, 2001, © all rights reserved.)

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Automatic Frequency Control

- **Stabilizes f_c against drift with T, [ppm/C]**
- **Change in drift with AFC and w/o AFC**

$$\Delta f_{c\ \mathrm{AFC}} = \frac{\Delta f_c}{1 + N_1 N_2 k_d k_o}$$

- **Raise stability by…**
  - **Raising N_1 and N_2**
  - **Raising k_d (discriminator slope, [v/Hz])**
  - **Raising k_o (VCO output/input slope, [Hz/V])**
- **Check out Example 6-9 on pp. 264-265 for application of formula**
  - **w/o AFC: drift of 1,020 Hz at VCO (2 ppm) becomes drift of 18,360 Hz at transmitter**
  - **w AFC: drift at transmitter reduced to 153 Hz**

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Automatic Frequency Control

- **Other causes of instability: discriminator drift (±100 ppm, typically), Δf_d, and reference oscillator drift, Δf_0**
  - **Numerator of closed-loop drift equation *with all drifts included*…**

$$\Delta f_{\text{total}} = N_1 N_2 \left(\Delta f_c + k_o k_d \Delta f_d + k_o k_d N_4 \Delta f_0 \right)$$

Note: Typo in Tomasi! Omits "Δ"

- **Total drift with AFC engaged…**

$$\Delta f_{c\ \text{AFC}} = \frac{N_1 N_2 \left(\Delta f_c + k_o k_d \Delta f_d + k_o k_d N_4 \Delta f_0 \right)}{1 + N_1 N_2 k_o k_d}$$
$$= \underbrace{\frac{N_1 N_2 \Delta f_c}{1 + N_1 N_2 k_o k_d}}_{\text{VCO drift}} + \underbrace{\frac{N_1 N_2 k_o k_d \Delta f_d}{1 + N_1 N_2 k_o k_d}}_{\text{Discriminator drift}} + \underbrace{\frac{N_1 N_2 N_4 k_o k_d \Delta f_0}{1 + N_1 N_2 k_o k_d}}_{\text{Xtal oscillator drift}}$$

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Direct FM VCO Transmitter w Phase-Lock-Loop (PLL) Freq Control

- **Message in (ac only) @ f_m**
- **Passed to VCO; out @ $f_t(t)$**
- **Output split**
  - **To FM out**
  - **To freq control**
    - **f_t divided by N to f_0**
    - **Compared to Xtal reference**
      - **Phase compared ($v_{out} \sim \Delta f$);**
    - **Slow dc correction signal generated and filtered**
    - **Added to signal to VCO; corrects center transmission frequency**

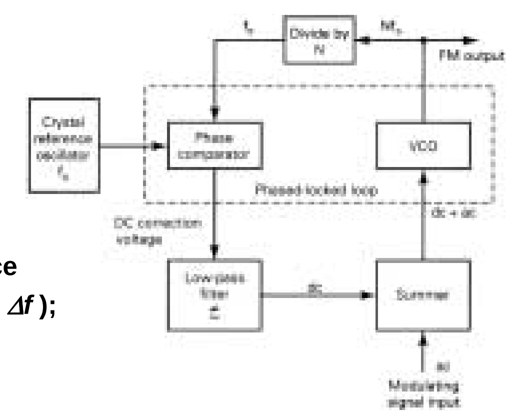

FIGURE 6-24 Phase-locked-loop FM transmitter (From Tomasi, 2001, © all rights reserved.)

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Armstrong Indirect FM Transmitter

- Signal in to balanced modulator (frequency upshift of 200 kHz from quadrature shifted Xtal oscillator signal); produces DSB SC signal
- Combined (vector added) to crystal in-phase oscillator signal; produces PM signal (with low index) at f_m with peak ~V_m

$$\theta = m = \tan^{-1}\left(\frac{V_m}{V_c}\right) \approx \frac{V_m}{V_c} \quad \text{for } V_m \ll V_c$$

- Result filtered, frequency-multiplied (72x)
- Upconverted on 13.15 MHz carrier
- Multiplied (72x)
- Transmitted

FIGURE 6-25 Armstrong indirect FM transmitter (From Tomasi, 2001, © all rights reserved.)

AM-to-PM at Combiner

a) Carrier phasor

b) Sidebands (at 90° to carrier due to phase shift)

c) Vector sum of carrier and sidebands

d) Max θ

e) Continuation of rotation

f) Zero angle (max $|V_c|$)

Angle $\theta = m \approx V_m/V_c$ (depends on amplitude, not freq = indirect FM)

Resultant $v(t)$: angle (really small) varies in time, follows $v_m(t)$

Note slight amplitude modulation of length of resultant (noise)

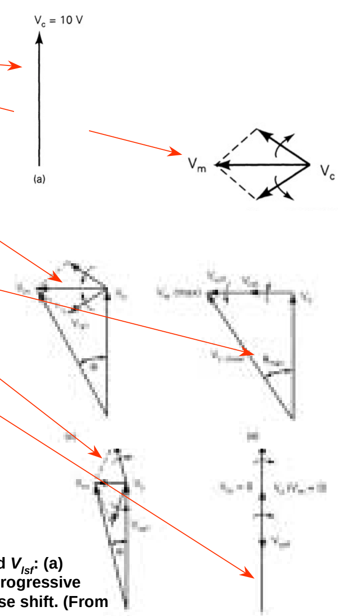


FIGURE 6-26 Phasor addition of V_c, V_{usf}, and V_{lsf}: (a) carrier phasor; (b) sideband phasors; (c)-(f) progressive phasor addition. Part (d) shows the peak phase shift. (From Tomasi, 2001, © all rights reserved.)

Example 6-10, p. 268

- V_c = 10 V, $V_{lsf} = V_{usf}$ = 0.048 V

a) $V_{m\,pk} = V_{usf} + V_{lsf}$ = 0.096

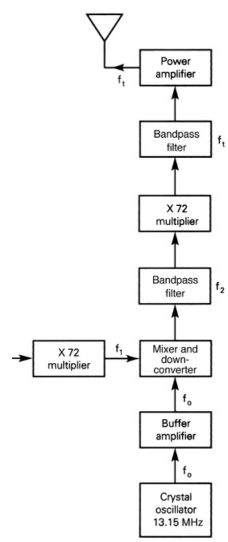
$\theta_{max} = m = 2(0.0096)/10 = 9.6\text{x}10^{-4}$ radians

b) $\Delta f = mf_m = (9.6\text{x}10^{-4})(15\text{ kHz}) = 14.4$ Hz (pretty small; begs for multiplication of 2508 to become bigger (75 kHz))

Multiplication and Upshifting

- Biggest typical $\theta = 1.67\text{x}10^{-3}$ radians
- $m \approx \theta = 1.67\text{x}10^{-3}$
- so $\Delta f_{max} = mf_{m\,max} = (1.67\text{x}10^{-3})(15\text{ kHz}) = 25$ Hz
- To fill allowed deviation of 75 kHz

  M = $75\text{x}10^3/25$ = 3,000!
- Too big. Use combination of upconversion and multiplication
- Multiply by 72: f_1, m, Δf all increase (f_1 = 14.4 MHz; m = 0.06912 rads; Δf = 1,036.8 Hz)
- Downconvert by 13.15 MHz (f_2 = 1.25 MHz; m = 0.06912 rads; Δf = 1,036.8 Hz)
- Multiply by 72: f_1, m, Δf all increase (f_t = 90 MHz; m = 4.98 rads; Δf = 74,650 Hz)
- Net effect: carrier increased by 450x; m, Δf by 5,184x



Make an FM Transmitter from PM

- **Add integrator before signal input**
- **Integrator = low-pass filter**
  - **Can be tricky if dc present in signal**

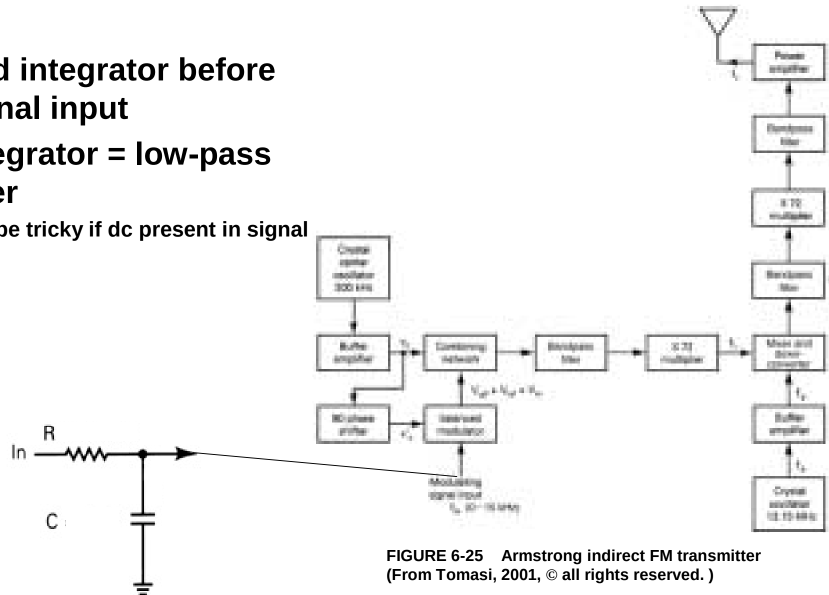


FIGURE 6-25 Armstrong indirect FM transmitter
(From Tomasi, 2001, © all rights reserved.)

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FM vs. PM

- ***m* is defined differently**
- $m_{FM} = K_1V_m/f_m = \Delta f/f_m$ ***(note dependence on frequency!)***
- $m_{PM} = KV_m = \Delta\theta$ **(no frequency dependence!)**

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Pros and Cons of Angle Modulation

- **Pros**
  - Immunity from amplitude-changing noise (e.g., atmospheric transmission)
  - Frequency capture effect- strongest signal dominates receiver (receiver "locks on" to the dominant signal)
  - Total power transmitted is constant, regardless of message
- **Cons**
  - Uses more bandwidth than AM
  - More complex (=$$) modulators and receivers

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Summary

- Angle modulation
  - FM
  - PM
- Time-domain equations (Table 6.1, p. 233)
- Parameters (Table 6.2, p. 237):
  - Message amplitude, frequency
  - Carrier amplitude, frequency
  - Frequency or phase deviation
  - Deviation sensitivity
  - Modulation index
  - Power
- Frequency domain
  - Sidebands
  - Bessel-function amplitudes
  - Bandwidth (nonnegligible amplitudes)
  - Power (total, sidebands)
- Noise interference (sinusoidal noise signal); FM demod reduces noise; increases SNR
- Preemphasis and deemphasis for larger SNR at upper frequencies (FM only)
- Direct FM modulators and transmitters
- Frequency upconversion (heterodyning and freq multiplication)
- Automatic frequency control (frequency stability)
- Indirect transmitter
  - Armstrong indirect FM transmitter
- Pros and cons of FM/PM

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